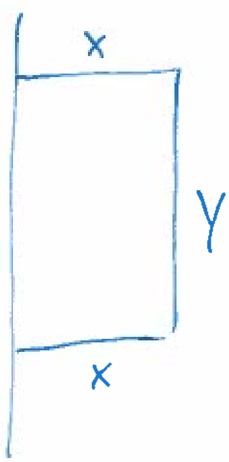
Solutions

Exam 3
Chapters 4.6-4.8 and 5

Short Answer
Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a $\boxed{\text{box}}$.

1. (10 points) You are trying to fence in a rectangular garden along the side of your house with 100 meters of fencing. Assuming that one of the sides of the garden is your house, what is the maximum area that you can fence in?



$2x + y = 100 \Rightarrow Y = 100 - 2x$

$\text{Area} = A = x \cdot Y = x(100 - 2x)$

$= 100x - 2x^2$

$A' = 100 - 4x$

crit. pt. at $x = 25$.

$A'' = -4$ so concave down

Thus $x = 25$ is a max

and $y = 50$.

Max Area $= 25 \cdot 50 = 1250\ m^2$

2. (10 points)Find $\int \frac{1}{x} + e^x - 12x^3 + \frac{1}{8}\sin(x)dx$.

$\ln|x| + e^x - 3x^4 - \frac{1}{8}\cos x + C$

3. (5 points) Find the area between the curve $f(x) = 3x^3 + 2x$ and the x-axis between $x = 2$ and $x = 5$.

$$\int_2^5 3x^3 + 2x\,dx = \frac{3}{4}x^4 + x^2 \Big|_2^5 = \left(\frac{3}{4}\cdot 5^4 + 5^2\right) - \left(\frac{3}{4}\cdot 2^4 + 2^2\right)$$

$$= \frac{1975 - 64}{4} = \frac{1911}{4}$$

4. (5 points) A pool is draining at a rate of $e^{-0.1t}$ litres per minute. How much water will drain from the pool in the first hour?

$$\int_0^{60} e^{-0.1t}\,dt = -\frac{e^{-0.1t}}{0.1}\Big|_0^{60} = -10(e^{-6} - e^{-0}) = 10(1 - e^{-6})$$

litres.

5. (10 points) A particle is moving along a straight line with acceleration at time t given by $a(t) = \cos(3\pi t) - \frac{1}{t^2}$ meters per second^2. Given that initial velocity $v(1) = -12$ meters per second and initial displacement $s(1) = 0$ meters, find the displacement for the particle as a function of t.

$$v(t) = \int a(t)\,dt = \frac{\sin(3\pi t)}{3\pi} + \frac{1}{t} + C$$

$$-12 = v(1) = \frac{1}{3\pi} + C \Rightarrow C = -13$$

$$v(t) = \frac{\sin(3\pi t)}{3\pi} + \frac{1}{t} - 13$$

$$s(t) = \int v(t)\,dt = \frac{-\cos(3\pi t)}{9\pi^2} + \ln|t| - 13t + C$$

$$0 = s(1) = \frac{1}{9\pi^2} + 0 - 13 + C \Rightarrow C = 13 - \frac{1}{9\pi^2}$$

$$s(t) = \frac{-\cos(3\pi t)}{9\pi^2} + \ln|t| - 13t + 13 - \frac{1}{9\pi^2}.$$

6. (10 points) Find $\int_0^5 x^2 e^{x^3}\,dx$.

$u = x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$u(5) = 125$

$u(0) = 0$

$$\frac{1}{3}\int_0^{125} e^u\,du = \frac{1}{3}e^u\Big|_0^{125} = \frac{1}{3}\left(e^{125} - e^0\right) = \frac{1}{3}\left(e^{125} - 1\right).$$

Bonus Question: (10 points) Find $\frac{dy}{dx}$ where

$$y = \int_0^x e^{t^2} + \ln(t^3 + 1)dt.$$

$\frac{dy}{dx} = e^{x^2} + \ln(x^3+1)$ by FTC.